Subprime Mortgages and Home Equity Lines of Credit: Theoretical Underpinnings from the Demand-Side

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Subprime mortgages, HELOCs, supply-side restrictions, fraud and misrepresentation have been postulated as causes of the “housing bubble” in the U.S. in the early- to mid- 2000s. This paper offers a theoretical demand-side explanation instead. Utilizing a sunspot model of housing demand and home equity lending, it is shown how agent preferences generate sunspot equilibria which cause housing prices to be excessively volatile. It is also suggested how the Fed’s dramatic reductions, then increases in interest rates during the early- to mid- 2000s, could have played a role in increasing housing price volatility. Finally, this paper shows how tax policy could be used to eliminate sunspots in the housing market. If this tax policy is not followed, housing price volatility could increase like in the U.S. and Japan (more than a decade earlier).

Field of Research: Real Estate Finance, General Equilibrium

1. Introduction

Subprime mortgages and home equity lines of credit (HELOCs), intended to increase homeownership and consumption levels, have been broadly blamed for the “housing bubble” in the United States in the early- to mid- 2000s. Other researchers (Vandell 2008) have provided other explanations: (1) subprime lending largely was displacing other loans that would have been made; (2) the problem with prices was primarily in the supply of new housing, not with the availability and cost of mortgage credit; (3) the problem was not subprime lending per se, but the Fed’s dramatic reductions, then increases in interest rates during the early- to mid- 2000s; (4) the housing “boom” was concentrated in markets with significant supply-side restrictions, which tend to be more price-volatile; and (5) the problem was primarily one of fraud and/or misrepresentation on the part of aggressive mortgage underwriters or borrowers, not in the presence of subprime lending per se. This paper offers a theoretical demand-side explanation for the “housing bubble,” and the subsequent crash and home equity lending losses. Shiller (2007b) has looked at a broad array of evidence, and has found that it does not appear possible to explain the housing boom in terms of fundamentals such as rents or construction costs. A psychological theory, that represents the boom as taking place because of a feedback mechanism or social epidemic that encourages a view of housing as an important investment opportunity, fits the evidence better.

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Lim (1997) introduced a simple general equilibrium model of housing demand to explain short-run booms and busts in the housing market. He suggested that speculation could cause cyclical movements around the fundamental long-run price, and such speculation arose from the existence of stationary sunspot equilibria. Stationary sunspot equilibria are multiple equilibrium paths around a steady state where the actual path, undetermined by fundamentals, is determined by nonfundamentals or sunspots. Hendershott et al. (2010) pointed out that due to nonrecourse mortgage loans in the United States, it is not the steady-state but the expectation of future appreciation around the steady-state that is relevant to the purchaser’s investment decision. Although sunspot prices are martingales and reflect all publicly available information, they also reflect extraneous information and are therefore excessively volatile. “‘Sunspots’ is short-hand for ‘the extrinsic random variable’ upon which agents coordinate their decisions, that is, one that does not affect economic fundamentals, but can affect economic outcomes. Sunspots are said to matter when the allocation of resources depends in a non-trivial way on the realization of the sunspot variable. Sunspot equilibria are instances of ‘excess volatility’. They arise even when expectations are fully rational ... Sunspot models are complete general equilibrium models that offer an explanation of excess volatility. It was by no means a new idea that economies can and do generate excess volatility, but the sunspots model is the first general-equilibrium model to exhibit excess volatility even when agents are fully rational.” (Shell 2008).

Lim’s (1997) model restricts mortgage lending based on the borrower’s ability to repay (for example, based on the borrower’s income). Instead of using “ability-to-repay”, this paper extends Lim’s (1997) model by incorporating home equity lending. Here, the borrower is able to borrow up to a fraction of the value of the home. As the home serves as collateral, unlike in the Lim (1997) model, the borrower is not restricted in borrowing by other measures of ability to repay (e.g., income). In some instances, borrowers borrow up to the value of their homes as no down payment is required. While this extension does not create problems with stable home prices, in period of bubbles and busts, this could create home equity lending losses as borrowers could end up owing more than the value of their homes, with no ability to repay their loans as incomes are insufficient.

In this model, it is shown how agent preferences generate sunspot equilibria which cause housing prices to be excessively volatile, and how this results in home equity lending losses. It is also suggested how the Fed’s dramatic reductions, then increases in interest rates during the early-to mid- 2000s could have played a role in increasing housing price volatility. In addition, financial contagion is examined (cross-country spillovers of housing price volatility) which has implications for emerging markets. Finally, the paper suggests how a certain tax policy could be used to eliminate sunspots in housing markets and possibly avert future mortgage crises. If this tax policy is not followed, housing price volatility could increase. The paper proceeds as follows: Section II presents related literature on the risk-premium puzzle in real estate and housing prices in utility functions, while Section III presents the sunspot model of housing demand and home equity lending. Applying this sunspot model to examine financial contagion and how monetary and fiscal policies interact with housing price volatility is presented in Section IV. Section V concludes with a discussion of the empirical evidence.
2. Literature Review

Shilling (2003) found that ex ante expected risk premiums on real estate were quite large for their risk, too large to be explained by standard economic models. Furthermore, the results suggested that ex ante expected returns were higher than average realized returns from 1988-2002, indicating that real estate experienced unexpected capital losses. This paper attempts to show that the divergence of expected and actual returns could be caused by extrinsic uncertainty or sunspots. As Shilling (2003, p.502) found that “investors appear to price all property types in the same way”, for simplicity, the model would only consider residential housing and ignore commercial real estate in order to focus on how investors form their expectations. The indeterminacy of equilibria in the model leads to “the fact that real estate investors appear to be no more uncertain about expected future returns after a decrease in price and fall in return than after an increase in price and return”. This indeterminacy may also explain Shiller’s (2007a) finding that the causes of turning points in real estate remain fuzzy. Shiller (2007b, p.7) argued that “a significant factor in this boom was a widespread perception that houses are a great investment and the boom psychology that helped spread such thinking”. These beliefs were compounded by a “burgeoning of real estate advertisements” (Shiller 2007b, p.20). Most people also mistakenly cited low interest rates, instead of expected rates of house price appreciation, as the main motivator of a good time to buy a house. “Money illusion” also appeared to be an important factor (Shiller 2007c).

Fisher et al. (2003), Goetzmann and Peng (2006) and Lin and Vandell (2007) have explained Shilling’s (2003) “risk premium puzzle” by real estate’s variable liquidity or significant time on market (TOM). Fisher et al. (2003) addressed the variable liquidity problem by developing the concept of a “constant-liquidity value” index for private asset markets, and found that constant-liquidity values tended to lead transaction-based and appraisal-based indices in time and also displayed greater volatility and cycle amplitude. Goetzmann and Peng (2006) showed that the ratio of sellers’ reservation prices to the market value affected trading volume and biased observed transaction prices as follows: when trading volume decreased (increased), index returns were estimated with an upward (downward) bias. Lin and Vandell (2007), basing on the notion that real estate transactions required costly searches, formulated a model to consider two distinct types of risk faced by the seller in real estate: marketing period risk and liquidation risk, and estimated the biases presented by these risks from the U.S. residential and commercial property markets. Both types of risks were positively correlated in their model. In the model, both types of risks (which are directly related to the seller’s reservation price) are proxied by including prices in the seller’s utility function. Having prices in the utility function generates sunspot equilibria and resulting pricing biases (like the “risk premium puzzle”).

Shell (2008) noted that sunspots could also arise from buyer search and associated non-convexities. One of the earliest stopping-rule search models was developed by MacQueen and Miller Jr. (1960). Turnbull and Sirmans (1993) applied this model to buyer search behavior as follows: because the selling price of each seller $p$ is not known until the buyer initiates contact, the buyer’s problem is to act as a price taker, searching from seller to seller, sampling repeatedly from the selling price distribution $f(p)$ until one price
is found to maximize the net gain from the entire search-purchase activity. Each sampling cycle is conducted at a cost $c$. The buyer’s problem is to find the optimal search stopping rule or reservation price $p^*$ which requires the buyer to continue searching until $p^*$ is found. Housing prices thus enter the utility function either through the net value of the house (which depends on $p$) or through leisure (which is reduced with longer search depending on $f(p)$).

Lim (1997) postulated other reasons for having housing prices in the utility function from the extant literature: (1) Generalized wealth effects in expected utility (Dusansky and Wilson 1993); (2) Endogenous consumption risks (Turnbull 1994); (3) Scitovsky effects (Scitovsky 1945; Wolinsky 1983); (4) Neighborhood effects or “location, location, location” (Veblen 1899; Samuelson 1972; Goodman 1989). The next section shows how prices in utility functions lead to sunspots.

3. Sunspot Model of Housing Demand and Home Equity Lending

Housing is assumed to be the only real asset and each agent is constrained to consume the same amount of owner housing that she has in her investment portfolio. There is a continuum of identical infinitely-lived agents who enter any given period $t = 1, 2, \ldots$ holding last period’s housing stocks $h_{t-1}$ and having to repay last period’s home equity loan or mortgage $l_{t-1}$. An amount $y_t$ of the (perishable) consumption good is endowed to each agent each period, and each agent owns shares of the exogenous mortgage or lending institution from which each agent receives a dividend $d_t$ of the consumption good each period. Each agent has the opportunity in period $t$ of taking out a home equity loan or mortgage $l_t$ at a discount of $q_t$. Each loan is repaid in period $t+1$ at face value, so the loan or mortgage rate (in terms of the consumption good) between periods $t$ and $t+1$ is $r_t = (1-q_t)/q_t$. Agents must decide how to allocate their endowment $y_t$, dividend $d_t$, housing wealth $h_{t-1}$ and new loan or mortgage $ql_t$ between current consumption $c_t$, housing $h_t$ and loan or mortgage repayment $l_{t-1}$.

Agents receive utility from the consumption good $c_t$ and housing $h_t$ as in Lim (1997). Housing $h_t$ consumed this period becomes part of an agent’s wealth next period, and thus housing is both a consumption and an investment good. More importantly, housing prices $p_t$ are also modeled in the utility function. $p_t$ is the relative price of housing in period $t$ in terms of the consumption good (or $1/p_t$ is the relative price of the consumption good $c_t$). The overall utility of consumption over all periods is given by discounting the stream of utilities (1) subject to each period’s budget constraint (2) and loan constraint (3). The loan constraint simply states that agents cannot borrow more than the value of their homes.

$$\max_{c_t, h_t, l_t} \sum_{t=0}^{\infty} \beta^t U(c_t, h_t, p_t)$$  \hspace{1cm} (1)

subject to

$$c_t + p_t h_t + l_{t-1} \leq y_t + p_t h_{t-1} + q_t l_t + d_t$$  \hspace{1cm} (2)

$$q_t l_t \leq p_t h_t$$  \hspace{1cm} (3)
Following Lim (1997), the within-period utility function $U$ is assumed to be twice-continuously differentiable, and strictly increasing and strictly concave in $c_t$ and $h_t$. $\beta$ is the discount factor, $0 < \beta < 1$. Given this setup, the optimality conditions for the agent’s problem can be found by considering the Lagrangean expression:

$$L = \sum_{t=0}^{\infty} \beta^t \{ U(c_t, h_t, p_t) - \lambda_t [c_t + p_t h_t + l_{t+1} - y_t - p_t h_{t+1} - q_t l_t - d_t] - \mu_t [q_t l_t - p_t h_t] \}$$  \hspace{1cm} (4)$$

with $c_t$, $h_t$ and $l_t$ as the choice variables. Because of the assumptions on $\beta>0$ and $U$ which assure that $c_t$ and $h_t$ will be strictly positive, the first-order Euler conditions associated with $c_t$, $h_t$ and $l_t$ can be written as equalities holding for all $t = 1, 2, \ldots$. They are, with $U_c$ denoting the partial derivative of $U$ with respect to consumption and $U_h$ denoting the partial derivative of $U$ with respect to housing:

$$U_c(c_t, h_t, p_t) - \lambda_t = 0$$  \hspace{1cm} (5)$$

$$U_h(c_t, h_t, p_t) - \lambda_t p_t + \beta \lambda_{t+1} p_{t+1} + \mu_t p_t = 0$$  \hspace{1cm} (6)$$

$$l_t [\lambda_t q_t - \beta \lambda_{t+1} - \mu_t q_t] = 0$$  \hspace{1cm} (7)$$

From equation (5), assumptions on $U$ imply a positive $\lambda_t$ in each period and therefore equation (2) holds with equality in each period. However, practical considerations restrict $q_t l_t < p_t h_t$ as it is almost always the case that the housing down payment is strictly positive. Therefore $\mu_t = 0$ (Kuhn-Tucker). three further simplifying assumptions are made. Clayton (1996) notes that the durability of housing implies that over the short term, the existing housing stock completely dominates any new supply. The housing problem is also synonymous with the land problem in Japan where the supply of land is fixed, as noted by Noguchi (1994) in discussing the housing bubble in Japan in the late 1980s. Thus housing supply is fixed at $h$.

A1. The supply of housing is fixed at $h$, $h_t = h$ for all $t$.

A2. Constant endowments, $y_t = y$ for all $t$.

A3. The exogenous lending institution makes loans as desired by agents (who are its owners in our representative agent general equilibrium setup). The revenue of this institution (per owner agent) is therefore $l_{t+1} - q_t l_t$, which is the loan repayment (with interest) from an agent less the new loan made to the agent. Let the cost of running this institution be fixed. This profit (revenue less cost) could be either paid as dividends or kept as retained earnings. Without loss of generality, It is assumed there are no retained earnings and zero fixed costs. The assumption of fixed costs implies that it will not affect the equilibrium properties of the model. Therefore, it is without loss of generality to set fixed costs equal to zero each period. Therefore:

$$l_{t+1} - q_t l_t = d_t$$  \hspace{1cm} (8)$$
Assumption A3 is not unrealistic as lending institutions were reported to be lending as much as borrowers wanted during the “housing bubble” in the early- to mid-2000s with little regard for creditworthiness. Hendershott et al. (2010) reported a zero-down program initiated by the United States Government in 2001 which enabled borrowers to bid and borrow for homes above what they could afford. Now Assumptions A1 to A3, together with the agent’s budget constraint equation (2), imply \( c_t = y_t = \mathbf{y} \) for all \( t \), which is the goods market equilibrium.

With the above assumptions, equations (3), (5), (6), (7) and (8) will determine equilibrium paths for \( l_t, \lambda_t, p_t, q_t \) and \( d_t \) respectively. From equation (5), \( \lambda_t = U_c(c_t, h_t, p_t), \lambda_{t+1} = U_c(c_{t+1}, h_{t+1}, p_{t+1}) \) and so forth. Constant endowments and housing supply imply that \( \lambda_t = U_c(\mathbf{y}, \mathbf{h}, p_t), \lambda_{t+1} = U_c(\mathbf{y}, \mathbf{h}, p_{t+1}) \) and so forth. Therefore \( l_t, p_t, q_t \) and \( d_t \) are determined by:

\[
q_d t < p_t \mathbf{h} \tag{3a}
\]

\[
U_t(\mathbf{y}, \mathbf{h}, p_t) - p_t \ U_c(\mathbf{y}, \mathbf{h}, p_t) + \beta \ p_{t+1} \ U_c(\mathbf{y}, \mathbf{h}, p_{t+1}) = 0 \tag{6a}
\]

\[
l_t[\ U_c(\mathbf{y}, \mathbf{h}, p_t) q_t - \beta \ U_c(\mathbf{y}, \mathbf{h}, p_{t+1})] = 0 \tag{7a}
\]

\[
l_{t+1} - q_d t = d_t \tag{8}
\]

The equilibrium time path of housing prices \( \{p_t\} \) is solely determined by equation (6a) as \( \{p_t\} \) is the only unknown in that equation. Equation (7a) then determines the evolution of the mortgage discount rate \( \{q_t\} \) where the mortgage interest rate \( r_t = (1 - q_t)/q_t \). As noted by Lim (1997), equation (6a) has the form \( G(p_t, p_{t+1}) = 0 \). It is first shown that there exists a steady state price \( p \) such that \( G(p, p) = 0 \) in equation (6a) and steady-state values of \( q_t, l_t \) and \( d_t \) can be derived from this steady state price \( p \) in the other three equations. It may then be shown that \( p_{t+1} = g(p_t) \) from equation (6a) and that \( g \) is a contraction for a nonempty, open set of economies, and finally construct stationary sunspot equilibria on housing prices for this set of economies. The volatility in housing prices \( \{p_t\} \) would lead to volatility in \( \{q_t\} \) and volatility and additional indeterminacies in \( \{l_t\} \) and \( \{d_t\} \).

**Proposition 3.1.** (Existence of Steady-State Equilibrium) With \( 0 < \beta < 1, \mathbf{h} > 0 \) and \( \mathbf{c} = \mathbf{y} > 0 \), there exists a steady-state equilibrium where the steady-state housing price \( p > 0 \) satisfies (6a) such that \( G(p, p) = 0 \), the steady-state housing loan or mortgage rate \( r = (1 - \beta)/\beta \) where \( q = \beta \), the steady-state housing loan amount \( l < ph/\beta \), and the steady-state dividend \( d = 1(1 - \beta) \).

**Proof of Proposition 3.1.** One first shows that there exists a steady-state housing price \( p > 0 \) which satisfies (6a) such that \( G(p, p) = 0 \). By the Implicit Function Theorem, a unique value of \( p \) can be found fundamentally as a function of \( \mathbf{y} \) and \( \mathbf{h} \) if the following condition holds:

\[
(1 - \beta) \ U_c(\mathbf{y}, \mathbf{h}, p) - (1 - \beta) \ p \ U_{cp}(\mathbf{y}, \mathbf{h}, p) + U_{hp}(\mathbf{y}, \mathbf{h}, p) \neq 0 \tag{9}
\]
The subscript p denotes the partial derivative with respect to the housing price and thus $U_{cp}$ denotes the cross-partial of $U$ with respect to the consumption good and the housing price and $U_{hp}$ denotes the cross-partial of $U$ with respect to housing and the housing price. By assumptions on $U$, even if (9) is degenerate (a necessary condition being $U_{cp}$ and $U_{hp}$ having the same sign), all one needs to do is to perturb the utility functions and (9) will no longer be degenerate. Hence the set of utility functions where (9) is degenerate is of measure zero. Having established the existence of a steady-state housing price $p > 0$, for $l_t > 0$, (7a) implies that $U_c(y,h,p) q_t - \beta U_{cp}(y,h,p) = 0$ or $q_t = q = \beta$ since $U_c(y,h,p) > 0$ by assumptions on $U$. Thus the steady-state housing loan or mortgage rate $r = (1-q)/q = (1-\beta)/\beta$. There will be a continuum of steady-state housing loan amounts $l$ which satisfy $l < ph/\beta$ in equation (3a), but for each $l$, there is a unique $d$ from equation (8) such that $d = 1/(1-q) = 1/(1-\beta)$.  

The Lim (1997) model is a special case where $l = 0$. In this case, equation (7a) does not tell us anything about the steady state value of $q$ as $q$ is irrelevant in Lim’s (1997) model with no home equity lending ($l = 0$). With $l = 0$, $d = 0$. Otherwise if $l > 0$, $d = 1/(1-\beta) > 0$, that is, the per agent steady-state dividend is simply the steady-state loan or mortgage interest received per agent each period. Notice that the Implicit Function Theorem in the above proof only implies that there is a unique steady state value of $p$. It does not imply that the equilibrium path of $\{p_t\}$ over time is unique. The following proposition shows that there exists a nonempty, open set of economies where it is possible to construct an equilibrium path around the steady state such that $p_t = p + \xi_t$, $p_{t+1} = p + \xi_{t+1}$, … for sufficiently small $\{\xi_t\}$. This means the equilibrium path of $\{p_t\}$ is locally non-unique or indeterminate, although the steady state $p$ is unique. The difference between a unique steady-state value and a unique equilibrium time path is worth emphasizing. The indeterminacy leading to sunspot equilibria is due to multiple equilibria where coordination failures or misperceptions could result in a time path that appears (non-fundamentally) stochastic.

Proposition 3.2. (Indeterminacy of housing prices $\{p_t\}$) There exists a function $p_{t+1} = g(p_t)$ which satisfies (6a). For a nonempty, open set of economies, there exist neighborhoods $\eta$ of $p$ and $\zeta$ of zero such that for all $\xi \in \zeta$, the mapping $g$ is a contraction on $\eta \times \eta$.

Proof of Proposition 3.2. By the Implicit Function Theorem, one can write $p_{t+1} = g(p_t)$ if and only if $G_2(p_t, p_{t+1}) \neq 0$, where:

$$G_2(p_t, p_{t+1}) = \beta [U_c(y,h,p_{t+1}) + p_{t+1} U_{cp}(y,h,p_{t+1})]$$

As argued in Proposition 3.1., even if (10) is degenerate (a necessary condition being that $U_{cp} < 0$), all one needs to do is to perturb the utility functions and (10) will no longer be degenerate. To show that $g$ is a contraction, one needs to show that $g'(p_t) < 1$ at the steady state $p$. By the Implicit Function Theorem,

$$g'(p_t) = -\frac{G_1(p_t, p_{t+1})}{G_2(p_t, p_{t+1})}$$

$$= \frac{1}{\beta} \frac{U_c(y,h,p) + p U_{cp}(y,h,p) - U_{hp}(y,h,p)}{U_c(y,h,p) + p U_{cp}(y,h,p)}$$

$$= 1$$

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There is clearly an open set of utility functions such that \( g'(p_t) < 1 \). For example, if \( U_{cp} > (<<) 0 \), then one just needs \( U_{hp} > (<) 0 \) and large enough in absolute value such that the numerator will be smaller than the denominator in absolute value. Hence \( g \) is a contraction. ■

That \( g \) is a contraction is sufficient for sunspots to matter in the equilibrium path of housing prices \( \{p_t\} \). Notice that if housing prices are not present in the utility function, then \( U_{cp} = U_{hp} = 0 \) and \( g'(p_t) = 1/\beta > 1 \) and the steady state is forward unstable and determinate, and sunspots do not matter for \( \{p_t\} \). But since there is a nonempty, open set of economies (or utility functions) such that \( g'(p_t) < 1 \), one can construct sunspots affecting \( \{p_t\} \) as follows. The nonfundamental stochastic optimization problem is:

\[
\max_{c_t, h_t, l_t} E_t \sum_{t=0}^{\infty} \beta^t U(c_t, h_t, p_t) \tag{12}
\]

s.t.

\[
c_t + p_t h_t + l_{t-1} \leq y_t + p_{t-1} h_{t-1} + q_t l_t + d_t \tag{2}
\]

\[
q_t l_t \leq p_t h_t \tag{3}
\]

where \( E_t \) is the conditional expectation given information on prices at period \( t \). Following the bootstrapping technique of Farmer and Woodford (1997) and Spear et al. (1990), one constructs a large family of equilibria by replacing (6a) with:

\[
U_{h}(y, h, p_t) - p_t U_{c}(y, h, p_t) + \beta \ p_{t+1} U_{c}(y, h, p_{t+1}) - \xi_t = 0 \tag{13}
\]

Where \( \xi_t \) is drawn from a stationary, i.i.d. distribution \( \psi \) such that \( \int \xi \ d\psi(\xi) = 0 \). Hendershott et al. (2010) noted that due to nonrecourse home mortgage loans, it is the expectation of future appreciation, not the mean expectation (which has been set to zero) that is relevant to the investment decision. Notice that the equilibrium condition (6a) has the form \( G(p_t, p_{t+1}, \xi_t) = 0 \). Since \( \{\xi_t\} \) is exogenous and thus independent of \( \{p_t\} \), the Implicit Function Theorem implies that one can write \( p_{t+1} = g(p_t, \xi_t) \) if and only if \( G_2(p_t, p_{t+1}, \xi_t) \neq 0 \), which holds from (10) in Proposition 3.2. \( p_{t+1} = g(p_t, \xi_t) \) is then the forecast function used.

**Proposition 3.3.** (Existence of sunspots in housing prices) There exists a nonempty, open set of economies exhibiting nontrivial stationary sunspot equilibria in \( \{p_t\} \).

**Proof of Proposition 3.3.** With Propositions 3.1 and 3.2 and the above discussion, it remains to show that there exists an invariant measure for the forecast function \( p_{t+1} = g(p_t, \xi_t) \). For any continuous function \( h(p_{t+1}, \xi_t) \), define for \( \xi_t \in \zeta \), \( \xi_t \) i.i.d.,

\[
P h(p_t, \xi_{t-1}) = \int_{\zeta} h[g(p_{t-1}, \xi_{t-1}), \xi_t] d\psi(\xi_t) \tag{14}
\]

With \( g \) a continuous function of its arguments, the transition operator \( P \) plainly takes continuous functions into continuous functions by the Implicit Function Theorem. Rosenblatt’s Theorem then states there exists an invariant distribution for the price
formation process \((p_t, \xi_{t-1})\). This distribution, together with the forecast function \(g\), constitutes a stationary rational expectations equilibrium (REE). Since the random variable \(\xi_t\) is non-degenerate, the equilibrium is stochastically nontrivial.

With \(\{p_t\}\) non-fundamentally stochastic, that is, \(p_t = p + \xi_t, p_{t+1} = p + \xi_{t+1}, \ldots\) for sufficiently small \(\{\xi_t\}\), equation (7a) shows that for \(l_t > 0\),

\[
q_t = \beta \frac{U_c(y, h, p + \xi_{t+1})}{U_c(y, h, p + \xi_t)}
\]

which means that \(q_t\) is likewise nonfundamentally stochastic and thus home loan or mortgage interest rates \(r_t = (1-q_t)/q_t\) are nonfundamentally stochastic. From (3a):

\[
l_t < (p + \xi_t) h \frac{U_c(y, h, p + \xi_{t+1})}{\beta U_c(y, h, p + \xi_t)}
\]

and so \(l_t\), besides being satisfied by a continuum of values, has an equilibrium time path \(\{l_t\}\) which is also nonfundamentally stochastic. Finally, given the equilibrium path of \(\{l_t\}\), from (8):

\[
d_t = l_{t-1} - \left[\beta U_c(y, h, p + \xi_{t+1})/U_c(y, h, p + \xi_t)\right] l_t
\]

which says that \(d_t\) will also be non-fundamentally stochastic. Home equity lending losses occur when the sunspot variables, \(\xi_{t+1}\) and \(\xi_t\), are such that \(d_t < 0\), for example, when \(\xi_{t+1}\) is such that \(U_c\) (in the numerator) is large. Even though our economy was fundamentally deterministic, sunspots were found to matter, and nonfundamental or excess volatility ensued. If this model had fundamental uncertainty, then with multiplicity of equilibria, coordination failures in expectations formation could result in nonfundamental price paths with higher volatility than what would have been generated by fundamentals alone. That the deviation of house prices from fundamentals is due to price dynamics rather than an overreaction to fundamentals is consistent with the empirical evidence of Fraser et al. (2008) who studied actual (real) house prices relative to fundamentals in New Zealand and found disparities between actual and fundamental real house prices, that is, the existence of real house price bubbles.

4. Implications of the Sunspot Model

4.1 Financial Contagion

The United States was not the only country that experienced a housing boom in the early- to mid-2000s and subsequent crash and mortgage crisis. Shiller (2007b) mentioned that this boom is unique in its pervasiveness. Dramatic home price booms since the late 1990s have been in evidence in Australia, Canada, China, France, India, Ireland, Italy, South Korea, Russia, Spain and the United Kingdom. The United Kingdom also suffered a crash and mortgage crisis around the same period as the United States. Northern Rock was Britain’s biggest casualty of the credit crunch and had borrowed about 26 billion pounds from the Bank of England since it requested emergency funds in September 2007. There were also expectations that Northern Rock would be nationalized. Amongst emerging markets, it was reported that the Mexican housing
market was in the midst of a boom which has attracted investment from United States pension funds like CALPERS. There appeared to be no prior example of such dramatic booms (and busts) occurring in so many places at the same time as from the late 1990s to more a decade later.

Spear (1989) showed how financial contagion resulting in an international credit crunch could result from stationary sunspots. First construct a pair of identical first-order sunspot equilibria on each country under the constraint that no trade occurs between them, as shown in Propositions 3.1, 3.2 and 3.3. Then use the pair of rational expectations equilibrium (REE) forecast functions (the $g$'s) obtained to solve for the sunspot variable (the $\xi$'s) in terms of the prices on each country. When the sunspot variable is substituted in terms of one country's price in the other country's forecast function, the sunspot variable is eliminated from the equilibrium pricing. This yields new forecast functions for each country that depend on each country's own prices and prices for the other country. Spear (1989) proved that this construction would show that these forecasts are, in fact, stationary rational expectations equilibrium (REE) forecasts, and proved the existence of a pair of invariant measures for the stochastic processes defined by the new forecast functions for a nonempty, open set of economies. When trade between countries is allowed, define an exchange rate as the ratio of housing prices in both countries. Under this exchange rate regime, no trade between countries is an equilibrium outcome. Therefore, the rational expectations equilibrium (REE) constructed under the assumption that there is no trade across countries would, in fact, be an equilibrium for the model in which trade is not constrained (Spear 1989). For this equilibrium, the other country's housing prices play the role of sunspot variables, so the uncertainty in the two-country model and resulting contagion are endogenous. Spear's (1989) construction of correlated sunspot equilibria could also be used to explain contagion within a country. Shiller (2007a, 2007b) mentioned that the housing boom in the early- to mid-2000s in the United States was a national event due to contagion within the country from an intense national media frenzy over booms in specific regions of the country. Hendershott et al. (2010) showed that the correlation of house prices in different regions of the United States reached a high of above 0.6 in 2008.

4.2 Monetary Policy

The housing loan or mortgage interest rate was endogenously determined in our Sunspot Model, where $r_t = (1-q_t)/q_t$ and:

$$q_t = \beta \frac{U_c(y,h,p_{t+1})}{U_c(y,h,p_t)}$$

If, instead, the housing loan or mortgage interest rate is exogenously determined, say by a central bank, then a sufficient condition to rule out sunspot equilibria is for the central bank to set the housing loan or mortgage rate $r_t = (1-\beta)/\beta$ for all $t$. This ensures that $q_t = \beta$ for all $t$, and thus $U_c(y,h,p_{t+1}) = U_c(y,h,p_t)$ for all $t$, which is only possible if $p_{t+1} = p_t = p$ for all $t$. Now if the central bank were to vary $r_t$ (and thereby $q_t$) over time, then it must be the case that $p_{t+1} \neq p_t$ over time, and housing prices would be (fundamentally) volatile. However, without an "anchor" for $q_t$, sunspots would matter, and there would be nonfundamental or excess volatility as well. This suggests that the Fed's dramatic
reductions, then increases in interest rates during the early- to mid-2000s could have played a role in increasing housing price volatility. Meltzer (1995) reported that M1 growth in Japan rose from 3.5% for 1982-1985 to 8.1% in 1985-1988. Meltzer also pointed out that since land is the most durable asset, the increase in M1 growth would increase the price of land. The evidence, however, is mixed. Taylor (2007) found that monetary policy deviations during 2002 to 2005 might have been the cause of the boom and subsequent bust in housing starts and inflation. But Shiller (2007b) pointed out that Taylor did not present an analysis of the model’s success in the period before 2000 and disputed Taylor’s findings. In Japan, Noguchi (1994) examined the “bubbles vs. fundamentals” argument and concluded that “the land price appreciation during the 1980s cannot be explained unless the bubble element is introduced” (p.11). Monetary policy therefore only acted as a catalyst for the Japanese housing price bubble in the late 1980s and the United States housing price bubble in the early- to mid-2000s.

4.3 Tax Policy

In this subsection, the method of Lim (1997) is used to derive a somewhat similar tax policy which mitigates sunspot equilibria. Let \( \tau_t \) be a tax rate on housing. The after-tax housing price in period \( t \) is then \((1-\tau_t)p_h\). If \( \tau_t > 0 \), then housing is taxed with the proceeds rebated lump-sum to households \( v_t = \tau_t p_h l_t > 0 \). If \( \tau_t < 0 \), then housing is subsidized and financed by a lump-sum tax on households \( v_t = \tau_t p_h l_t < 0 \). The optimization problem then becomes:

\[
\max_{c_t, h_t, l_t} \sum_{t=0}^{\infty} \beta^t U(c_t, h_t, (1-\tau_t)p_h) \tag{19}
\]

s.t.

\[
c_t + (1-\tau_t)p_h l_t + l_{t-1} \leq y_t + (1-\tau_t)p_h l_{t-1} + q_t + d_t + v_t \tag{20}
\]

\[
q_t l_t \leq (1-\tau_t)p_h l_t \tag{21}
\]

With assumptions A1 to A3, the aforementioned assumptions on \( U \), let the tax rate \( \tau_t = \tau \) for all \( t \). Thus \( v_t = \tau_t p_h \). The remaining unknown variables \( l_t, p_t, q_t \) and \( d_t \) are determined by equations (22), (23), (24) and (8):

\[
q_t l_t < (1-\tau)p_h \tag{22}
\]

\[
U_h(y + \tau_t p_h h, (1-\tau_t)p_h) - (1-\tau)p_t U_c(y + \tau_t p_h h, (1-\tau_t)p_h) + \beta(1-\tau)p_{t+1} U_c(y + \tau_t p_{t+1} h, (1-\tau_t)p_{t+1}) = 0 \tag{23}
\]

\[
l_{t+1} - q_t l_t = d_t \tag{8}
\]

Given \( U, \tau, y, h \) and \( \beta \), equation (23) determines the time path of \( \{p_t\} \) as it is the only unknown variable in that equation. Having determined \( \{p_t\} \), equation (24) determines \( \{q_t\} \), and a continuum of values should satisfy \( \{l_t\} \) in equation (22). With an \( l_t \), equation (8) determines \( \{d_t\} \). A steady-state \( \mathbf{p} \) exists which fundamentally depends on \( U, \tau, y, h \) and \( \beta \).
This can be demonstrated following Proposition 3.1. What is more interesting is the derivation in the following proposition of a constant tax rate $\tau$ which will make the steady state determinate and eliminate sunspot equilibria.

**Proposition 4.1.** (Existence of tax policy to eliminate sunspots) There exists a constant tax rate $\tau = U_{hp}/(U_{hp} - U_{ch}h)$ (where $U_{hp}$ and $U_{ch}$ are evaluated at the steady-state housing price $p$) such that $g(p_i) > 1$ for all $U$, $\tau$, $y$, $h$ and $\beta$.

**Proof of Proposition 4.1.** By the Implicit Function Theorem, we can write $p_{t+1} = g(p_t)$ if and only if $G_2(p_t, p_{t+1}) \neq 0$, where:

$$G_2(p_t, p_{t+1}) = \beta(1-\tau)^*$$

$$[U_c(y + \tau p_t, h, h, (1-\tau)p_{t+1}) + p_{t+1}[\tau h U_{cc} (y + \tau p_t, h, h, (1-\tau)p_{t+1}) + (1-\tau)U_{cp} (y + \tau p_t, h, h, (1-\tau)p_{t+1})]]$$

(25)

As argued in Proposition 3.1., even if (25) is degenerate, all one needs to do is to perturb the utility functions and (25) will no longer be degenerate. To find a $\tau$ such that $g$ is never a contraction, one needs to show how $g'(p_i) > 1$ at the steady state $p$. By the Implicit Function Theorem,

$$g'(p_i) |_{p} = -\frac{G_1(p,p)}{G_2(p,p)}$$

$$= \frac{1}{\beta} \frac{(1-\tau)[U_c + p_c(\tau h U_{cc} + (1-\tau)U_{cp})] - [\tau h U_{ch} + (1-\tau)U_{hp}]}{(1-\tau)[U_c + p_c(\tau h U_{cc} + (1-\tau)U_{cp})]}$$

(26)

where the partial (and second and cross-partial) derivatives of the utility function are evaluated at the steady state $p$. Clearly a sufficient condition for $g' > 1$ is that $g' = 1/\beta$, which can be achieved by setting $\tau h U_{ch} + (1-\tau)U_{hp} = 0$, or $\tau = U_{hp}/(U_{hp} - U_{ch}h)$. ■

Now housing and prices are more likely to enter multiplicatively positively due to generalized wealth effects as postulated by Dusansky and Wilson (1993) or to Veblen or Scitovsky effects. With $U$ assumed strictly concave in $h$, $U_{pp} < 0$ implies that $U_{hp} < 0$. (It can be shown that even if $U_{hp} > 0$, the tax policy to mitigate sunspots will still be a net tax to housing, unless the complementary effect is sufficiently strong.) With $h > 0$, this means that the sign of $\tau$ depends on the sign of $U_{ch}$. If $U_{ch} > 0$, then the numerator and denominator of $\tau = U_{hp}/(U_{hp} - U_{ch}h)$ are both negative, so $\tau > 0$. Even if $U_{ch} < 0$, if $U_{ch}h < U_{hp}$, the numerator and denominator of $\tau = U_{hp}/(U_{hp} - U_{ch}h)$ will still both be negative, so $\tau > 0$. Only if $U_{ch} < 0$ and $U_{ch}h > U_{hp}$ in absolute values would the denominator of $\tau = U_{hp}/(U_{hp} - U_{ch}h)$ be positive and so $\tau < 0$. The latter condition differs from Lim (1997), who only required $U_{ch} < 0$ for $\tau < 0$. In our Sunspots Model, if housing and the other consumption good are Auspitz-Lieben complements, then housing should be taxed. Newman (1987) defined two commodities $x$ and $y$ as Auspitz-Lieben complements (substitutes) if the cross partial of the utility function $U_{xy} > 0$ ($U_{xy} < 0$). Even if housing and the other consumption good are Auspitz-Lieben substitutes, housing should be taxed unless the substitution effect is sufficiently strong. That is, with home equity lending, a housing tax would generally curb nonfundamental or excess volatility in housing prices. The need to tax housing speculation is stronger in this model than in Lim (1997) because
speculators are no longer constrained by income or wealth. As they could borrow up to the value of the homes, a housing tax is necessary to discourage excessive borrowing. Now if a different tax policy is followed, for example if housing is subsidized but the substitution effect is not sufficiently strong, then sunspots would be more likely to occur and so the housing subsidy would increase speculation and decrease social welfare for risk averse agents by Jensen’s inequality (Shell 2008).

5. Conclusion and Empirical Evidence

In this paper, a simple infinitely-lived agent model of housing and home equity lending, where housing is both a consumption and an investment good, is developed to examine short-run housing price volatility. It suggests that speculation could cause cyclical movements around the long-run trend (which is fundamentally determined) and could lead to home equity lending losses. Such speculation arises from the existence of stationary sunspot equilibria. Spear (1989) showed how stationary sunspot equilibria could spill over from one country to another, as one country’s price could serve as the sunspot for the other country. Thus nonfundamental or excess volatility in one country’s housing prices could lead to financial contagion resulting in an international credit crunch. However, sunspots are not ubiquitous and can often be mitigated by appropriate government policies. The results suggest that the central bank keep monetary policy steady. Dramatic changes in monetary policy could generate extrinsic uncertainty or sunspots, which lead to nonfundamental or excess volatility in housing prices. Meltzer pointed out that the dramatic increase in money supply in Japan in the late 1980s fueled the housing price bubble. However, Noguchi (1994) examined the “bubbles vs. fundamentals” argument in Japan and concluded that “the land price appreciation during the 1980s cannot be explained unless the bubble element is introduced” (p.11). Shiller (2007b) also found that monetary policy did not come out as central in his case studies of housing booms and busts. Monetary policy therefore acted only as a catalyst for housing bubbles worldwide.

The underlying cause of the Japanese housing price bubble during the 1980s was extrinsic. The major cultural factor was that the Japanese regarded a house as an asset that produced capital gains. It was said that they would buy a house to own rather than to live. Hulme (1996) called this a “land myth” – the pernicious notion that real estate prices could never go down. Compounding the belief that prices could only go up was a sense of limited supply. The “land myth” was a liquidity catalyst, a means to borrow money based on speculation. Between 1984 and 1989, total bank lending grew an average of 9.2% a year, while lending related to real estate grew at a rate of 20% a year. The Japanese housing price bubble was thus amplified by careless lending in the banking sector as financial institutions began “selling money” (Yamamuro 1996). This resulted in home equity lending losses. The late 1980s housing bubble in Japan is certainly similar to the housing price bubbles in the United States and other countries.

Lim (1997) derived a tax policy whereby there should be a net tax (subsidy) to housing if housing and other consumption are Auspitz-Lieben complements (substitutes). With home equity lending where borrowers are only limited by the value of the homes they buy, it is found that unless housing and other consumption are strong Auspitz-Lieben
substitutes, there should be a net tax on housing to curb speculation and thereby mitigate non-fundamental or excess volatility in housing prices. Lim (1997) estimated from Japanese data that housing is likely to be complementary to consumption, and he suggested that the complementary effect of housing is likely to hold in most countries. Therefore, he suggested that a heavier burden on housing and real estate taxes would decrease speculation. This could be implemented by raising the assessments for property taxes (which Ito (1994) said is what “all economists in Japan recommend”), a cautious increase in capital gains taxes for real estate transactions, and a landholding tax to raise the cost of holding land for speculation. The last suggestion was implemented by the Japanese Government in 1992 to curb short-term property price speculation.

The U.S. Taxpayer Relief Act of 1997 exempted the first $500,000 in capital gains from any home sale when the home is held for only two years or more. Several commentators (Bajaj and Leonhardt, 2008) have conjectured that it may have helped cause the housing bubble. Now this tax change reduced the holding period to two years for a $500,000 capital gain exemption, which represents a net subsidy to housing, in particular short-term housing price speculation. Lim (2011) examined housing price volatility before and after 1997 utilizing Cass-Shiller-Weiss quarterly national home price index and U.S. Federal Housing Finance Agency (formerly OFHEO) quarterly HPI (house price index). His results suggested that national housing price volatility increased after the U.S. Taxpayer Relief Act of 1997, and that the housing price bubble in the United States was caused by this net subsidy to housing. The housing subsidy generated excess or non-fundamental volatility in housing prices, and subsequent home equity lending losses which precipitated a mortgage crisis. By placing housing in a special privileged category for capital gains tax purposes, excess volatility in housing prices ensued. To reduce housing price volatility, this paper suggests that the holding period for the capital gain exemption for housing be increased to at least 5 years, perhaps even as long as 10 years (with certain allowances for job relocation).

In conclusion, a theoretical demand-side explanation for the “housing bubble” in the United States in the early- to mid-2000s, and the subsequent crash and mortgage crisis, has been presented. The major contribution of this paper could be in its tax policy recommendation: to mitigate nonfundamental or excess volatility in housing prices, there should generally be a net tax on housing speculation (similar to a “Tobin tax” on currency speculation). This tax would increase welfare by reducing housing price volatility for current and future homeowners. In particular, taxes on short-term housing capital gains should increase. The increase in housing price volatility in the U.S. and Japan (more than a decade earlier) caused by misguided tax policies could be useful lessons for developing countries facing renewed property price speculation.

Endnotes

1. Comments from David Brasington, Andrea Heuson, Robert Martin and Joseph Nichols, and participants at the Financial Management Association, American Real Estate and Urban Economics Association and the Asian Real Estate Society are appreciated. All errors remain my own.

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