

Applying Quantile Regression to Industry Default Risk in Europe

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The problems experienced by banks worldwide during the Global Financial Crisis (GFC), including bank failures and capital shortages, demonstrated the importance of understanding extreme credit risk in dynamic economic circumstances. We apply a novel Quantile Monte Carlo (QMC) model to measure extreme risk of various European industrial sectors both prior to and during the Global Financial Crisis (GFC). The QMC model involves an application of Monte Carlo Simulation and Quantile Regression techniques to the Merton structural credit model. Two research questions are addressed in this study. The first question is whether there is a significant difference in distance to default (DD) between the 50% and 95% quantiles as measured by the QMC model. A highly significant difference in DD between the two quantiles was found. The second research question is whether relative industry risk changes between the pre-GFC and GFC periods at the extreme quantile. Changes were found with the worst deterioration experienced by Energy, Utilities, Consumer Discretionary and Financials; and the strongest improvement shown by Telecommunication, IT and Consumer goods. As a robustness check the QMC model is compared to a published Conditional Value at Risk (CVaR) based model. Overall, we find a significant increase in credit risk for all sectors using the QMC model as compared to the traditional Merton approach. These findings could be important to banks and regulators in measuring and providing for credit risk in extreme circumstances.

Jel Codes: G01, G21, G28

1. Introduction

The extreme credit risk experienced by banks during the GFC was a stark reminder of the importance of measuring and providing for potential credit defaults in extreme circumstances. Many prevailing credit models, however, are based on 'average' or 'median' risk over a specified period, or in the case of Value at Risk (VaR) models, on risks below a selected threshold. The problem with this approach is that it does not capture the most extreme economic circumstances in which firms are most likely to fail. The Merton (1974) model, modified by KMV (Crosbie & Bohn, 2003) measures Distance to Default (DD) based on the standard deviation of market asset value fluctuations. We modify the Merton model using a combination of Monte Carlo Simulation and Quantile Regression to compare DD at different risk quantiles. The revised model is hereafter referred to as the Quantile Monte Carlo (QMC) model.

We apply the QMC model to measure DD of European industries both prior to and during the GFC. We consider Europe a particularly important setting in which to measure extreme credit risk, given the instability experienced by European financial markets and particularly the European banking sector during the GFC and subsequent sovereign debt crisis.

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Allen, Boffey & Powell

The first question addressed by this study is whether asset volatility (and the associated DD) is significantly different between median and extreme levels. Using the QMC model we measure this difference at the 50% and 95% quantiles. Secondly, we use the QMC model to investigate whether relative extreme industry risk is different between the pre-GFC and GFC periods. That is, were the most risky industries pre-GFC also the most risky during the GFC?

The study finds the difference between risk quantiles to be highly significant, suggesting that traditional approaches to credit risk fail to capture extreme events. This could lead to a shortfall in key risk management tools, such as provisions and capital, just when they are most needed. In addition, the study finds that relative industry risk changes during the GFC as compared to pre-GFC. These findings can be important to lenders and regulators in making decisions on credit portfolio mix, provisions and capital allocation.

Whilst the measurement of extreme risk is becoming increasingly prevalent in the measurement of share market risk, particularly since the Global Financial Crisis, studies of extreme measures for credit risk are comparatively very few and primarily centre around transition matrix methodology. Transition matrices measure the probability of an obligor transitioning from one credit grade to another, including the probability of transition to a default grade. CreditMetrics is a popular credit model which uses transition matrices to measure Value at Risk (VaR) which is the maximum anticipated loss at a selected threshold. The most notable work in regards to extreme credit risk using transition matrices is the application of Conditional Value at Risk (CVaR) to credit risk portfolios by Andersson, Mauser, Rosen and Uryasev (2000) and Uryasev and Rockafellar (2000). CVaR is the average of the losses beyond VaR. If VaR is measured at 95%, then CVaR is measured as the average of the worst 5% losses. Lucas, Klaasen, Spreij and Straetman (2002) also use CreditMetrics transition methodology to measure extreme tail behaviour of credit portfolios. A problem with transition matrices is that they are based on credit ratings, which do not ratchet up and down with changing economic circumstances. Credit ratings are often reviewed change only annually, or less frequently, and while this is generally fine during times of economic stability, these models were found to significantly understate credit risk during the highly volatile times of the GFC (Allen, Kramadibrata, Powell, & Singh, 2011a; Allen & Powell, 2011; Allen & Powell, 2012). The Merton-KMV Structural model used in this study, on the other hand, incorporates daily asset value fluctuations allowing it to respond much quicker to changing circumstances than ratings based models.

There have been other studies on sectoral risk in Europe (e.g., Allen, Powell, & Singh, 2011), but using VaR and CVaR based techniques as compared to the quantile regression techniques of this study. The innovative QMC model is new and unique to the authors. QMC techniques have only previously been applied in a few studies of bank and corporate risk in Japan, Australia and the US (viz. Allen, Kramadibrata, Powell, & Singh, 2010, 2011, 2011a) and have not been applied to sector comparisons or to any European studies. Thus, this paper makes a unique contribution in this regard.

The literature survey in section 2 provides a brief overview of the risk climate in Europe brought about by the GFC and sovereign debt crisis, as well as a summary of relevant credit risk literature. Methodology and data are discussed in Section 3, followed by findings and discussion in Section 4 and conclusions in Section 5.

2. Background and Literature Review

A combination of the GFC and the associated sovereign debt crisis has led to a prolonged period of financial instability in European markets. As happened in the US and other Global markets, the GFC led to a need in Europe for financial sector support programmes and macroeconomic stimulus to be provided by Central banks and Governments. The 2008 UK Government £500bn financial support package is one example. Another example is the liquidity support measures provided by the Bank of England (BOE) and European Central Bank (ECB), which included extension of maturity terms on refinancing operations and allowing banks to swap illiquid securities for liquid ones. Following an emergency Paris summit in 2008, Euro-area governments provided co-ordinated support measures to banks such as increasing deposit insurance, providing guarantees on bank bond issues and making capital injections into banks. Asset relief measures were introduced to remove or insure toxic bank assets (European Central Bank, 2009).

In 2010, concern that a number of European countries, including Greece, Portugal, Spain, Italy and Ireland would default on high debt levels led to a sovereign debt crisis. Comprehensive rescue packages by the International Monetary Fund and other Eurozone countries were formulated. The ECB also introduced measures to reduce volatility in financial markets and improve liquidity, such as commencing open market operations by buying government and private debt securities.

These events all provide a climate of extreme risk for investors and banks, leading to the need for accurate measurement of extreme risk. Techniques such as VaR, which measures risk below a predetermined threshold, have come under criticism. VaR, for instance, has undesirable mathematical properties; such as lack of sub-additivity (Artzner et al., 1999; 1997). But perhaps the biggest shortcoming of VaR is that it is focused on risks below a specified threshold and says nothing of the risks beyond VaR. The measurement has been criticized by Standard and Poor's analysts (Samanta, Azarchs, & Hill, 2005) due to VaR being applied inconsistently across institutions, as well as lack of tail risk assessment. This study provides a mechanism for measuring that tail risk.

The QMC model applied here modifies the Distance to Default (DD) structural approach of Merton (1974) and KMV (Crosbie & Bohn, 2003). The Merton / KMV model measures DD based on a combination of fluctuating asset values and the debt / equity (leverage structure) of the borrower. Examples of studies using structural methodology for varying aspects of credit risk include asset correlation (Cespedes, 2002; Kealhofer & Bohn, 1993; Lopez, 2004; Vasicek, 1987; Zeng & Zhang, 2001), predictive value and validation (Bharath & Shumway, 2008; Stein, 2007), fixed income modelling (D'Vari, Yalamanchili, & Bai, 2003), and effect of default risk on equity returns (Chan, Faff, & Kofman, 2008; Gharghori, Chan, & Faff, 2007; Vassalou & Xing, 2004). Besides fluctuating assets, the other key component of structural modelling is the borrower's leverage ratio. Leverage ratios of banks have come under close scrutiny during the financial crisis, with many requiring additional capitalisation. The leverage ratios in this study range from 5.3% for Financials to 53% for Health Care. The importance of fluctuating asset values in measuring credit risk has been raised by the Bank of England (BOE, 2008), who report that during the GFC "system-wide vulnerabilities were exposed...rooted in uncertainties about the value of banks assets...amplified by excessive leverage". As probabilities of default increase, there is greater likelihood of assets needing to be liquidated at market prices. BOE express a need for market participants to revalue their assets with greater

Allen, Boffey & Powell

weight placed on mark-to-market values. This gives rise to reduced asset values and a need for increased capital. Our model addresses the issue of measuring these extreme asset value fluctuations, with detailed methodology provided in the following section.

3. Methodology and Data

Under Merton structural methodology, the firm defaults when asset values fall below debt levels. Moody's KMV model (Crosbie & Bohn, 2003) is based on the Merton model, and is widely used by banks to measure Distance to Default (DD) from which associated Probability of Default (PD) can be calculated using a normal distribution assumption. Based on the thousands of defaulted firms in their worldwide database, KMV find that DD is most accurately measured when debt is taken as the value of all short-term liabilities (one year and under) plus half the book value of all long term debt outstanding. This is the approach used in this study. DD and PD are calculated as follows:

$$DD = \frac{\ln(V/F) + (\mu - 0.5\sigma_v^2)T}{\sigma_v\sqrt{T}} \quad (1)$$

$$PD = N(-DD) \quad (2)$$

Where V is the market value of the firm, F = face value of firm's debt, μ = an estimate of the annual return (drift) of the firm's assets, σ_v = standard deviation of the asset returns and T is usually set as 1 year.

A key component of determining DD is the calculation of the asset returns. Our data includes 10 years of daily returns for all S&P Euro stocks. This data, together with balance sheet items required to calculate DD, is obtained from Datastream. Data is split into GFC (2007-2009) and pre-GFC (2000-2006) periods. The pre-GFC period aligns with requirements for Basel advanced credit risk models. Following Merton and KMV methodology, using equity returns and the relationship between equity and assets, we estimate an initial asset return for each company in our data set. Crosbie & Bohn (2003) and Bharath & Shumway (2008) provide a detailed explanation of this methodology. Daily log return is calculated and new asset values estimated for every day. This is repeated until asset returns converge. Industry returns are calculated as the asset-weighted average of company returns in that industry. We do not calculate correlations as we are not calculating returns for investment purposes.

It is at this stage that we depart from the Merton model. In measuring these returns, we could just use historical returns as used by Merton and KMV. However, our extreme quantile (as explained below) is only based on a small portion of returns (the 5% tail), and Monte Carlo is a particularly useful tool in enriching the database when dealing with smaller numbers of returns. We generate 20,000 scenarios of returns, a similar number to that suggested by other prominent Monte Carlo Simulation studies (E.g., Uryasev & Rockafellar, 2000). These simulations are obtained by creating random numbers based on the distribution (mean and standard deviation) of historical returns. The daily asset weighted average of returns is calculated for each industry.

We then apply quantile regression, which as per Koenker & Bassett (1978) and Koenker and Hallock (2001) is a technique for dividing a dataset into parts. Minimising the sum of symmetrically weighted absolute residuals yields the median where 50% of observations fall either side. Similarly, other quantile functions are yielded by minimising the sum of

Allen, Boffey & Powell

asymmetrically weighted residuals, where the weights are functions of the quantile in question per Equation 3. This makes quantile regression robust to the presence of outliers.

$$\min_{\varepsilon \in R} \sum p_r(y_i - \varepsilon) \quad (3)$$

where $p_r(\cdot)$ is the absolute value function, providing the r^{th} sample quantile with its solution.

β is calculated at each quantile for the regression variables (for example returns of an individual entity against market returns).

We regress the 20,000 Monte Carlo simulated returns for each industry (split into our two time periods) against the Monte Carlo returns for all industries (the portfolio ‘benchmark’) for the 10 year period, and derive β at the 50% quantile (which yields similar results to the standard Merton model) and 95% quantile (our extreme returns). From this β we are able to calculate DD. Our regression is benchmarked on portfolio returns, and therefore β at the median (50% quantile) of portfolio returns is 1. The denominator of the DD calculation in Equation 1 is based on asset value fluctuations, and so is the β generated by the quantile regression. A change in β will, therefore, result in a proportionately equal change in DD. For example, β of 2 for an industry (i) at a particular quantile (q) and time period (t) yields a DD equal to half the benchmark (b) DD:

$$DD_{igt} = DD_b / \beta_{igt} \quad (4)$$

4. Results and Analysis

Table 1 shows DD and β for each period and quantile.

Table 1: Summary of Results

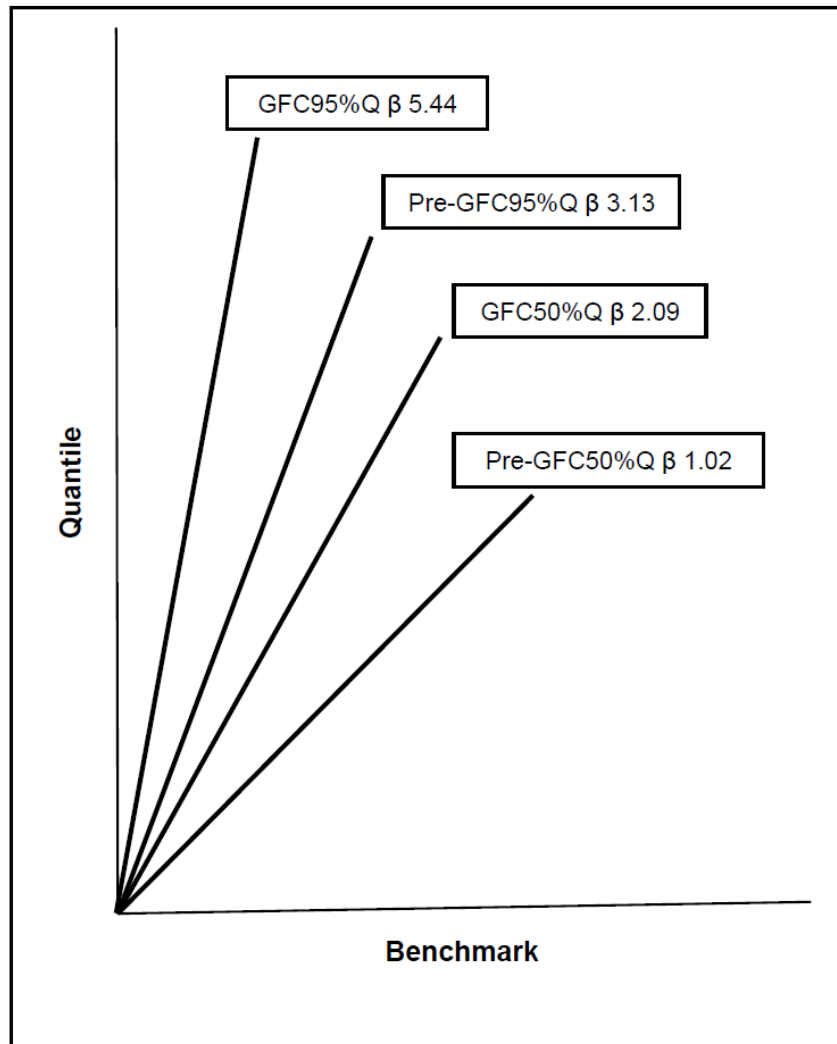
	DD						Beta (β)					
	Pre GFC		GFC		Total Period		Pre GFC		GFC		Total Period	
	50%Q	95%Q	50%Q	95%Q	50%Q	95%Q	50%Q	95%Q	50%Q	95%Q	50%Q	95%Q
Cons. Disc.	4.80	1.42	2.82	0.87	4.06	1.97	0.92	3.09	1.56	5.06	1.08	2.24
Cons. Stap.	5.15	1.43	4.12	1.24	4.75	2.96	0.86	3.09	1.07	3.55	0.93	1.49
Energy	5.11	1.70	2.97	0.95	4.54	2.12	0.86	2.59	1.48	4.64	0.97	2.08
Financials	4.30	1.41	2.10	0.81	3.60	1.45	1.02	3.13	2.09	5.44	1.22	3.04
Health Care	4.88	1.45	4.07	1.20	4.59	2.90	0.90	3.04	1.08	3.67	0.96	1.52
Industrials	5.32	1.60	3.33	1.08	4.79	2.32	0.83	2.75	1.32	4.09	0.92	1.90
IT	3.59	1.17	2.94	0.99	3.46	2.11	1.23	3.76	1.50	4.45	1.27	2.09
Materials	4.79	1.27	3.22	0.98	4.25	2.26	0.92	3.48	1.37	4.50	1.04	1.95
Telecomm.	4.23	1.15	3.99	1.69	4.30	2.75	1.04	3.82	1.10	2.61	1.02	1.60
Utilities	6.88	1.95	3.25	1.02	5.73	2.18	0.64	2.26	1.35	4.32	0.77	2.02
Total	4.90	1.45	3.28	1.08	4.41	2.30	0.90	3.03	1.34	4.07	1.00	1.91

Organisation of the tables is as follows: left half is DD and right half is Beta; the columns of each of the two halves are then organised into pre-GFC, GFC and Total Period; rows are sorted alphabetically rather than on value order given that value order is different for each column. DD (measured by number of standard deviations) is calculated at the 50% quantile (50%Q) and 95% quantile (95%Q) using Equation 1. Pre-GFC incorporates the 7 years to 2006. The GFC period is 2007-2009. Beta measures risk at each quantile relative to the 50%Q Total Period Quantile (shown in bold) per Equation 3.

Allen, Boffey & Powell

The first question addressed by this study is whether there is a significant difference in DD between the 50% and 95% quantiles as measured by the QMC model. We note from Table 1 that, across the board for all industries, there is substantial difference in DD between the two quantiles. Using an F test, we find these differences between the quantiles to be significant at the 99% level for all industries. As an example, GFC DD for the financial industry was 2.09 at the 50% quantile, which has a PD of 2% per Equation 2. But during the worst 5% of asset value fluctuations (95% quantile), DD reduced to 0.81 (due to a combination of high volatility and high leverage). This is precariously close to default with a PD of 21%. The difference between the quantiles for the financial industry is illustrated in Figure 1. We see that at the 95% quantile, the financial industry has a β of 5.44 (based on a DD of 0.81 being 5.44x higher than the portfolio benchmark DD of 4.41) as compared to a β of 2.09 at the 50% GFC level. This means that at the period of the worst asset value fluctuations of the GFC, default risk was substantially higher than indicated by the traditional DD measure. The average β of 4.07 at the GFC 95%Q indicates that credit risk was fourfold higher at the extremes of the GFC than at the total period benchmark. This makes intuitive sense, when considering that impaired assets (problem loans) for European banks also increased nearly fourfold from an average of 1.8% over the 5 years preceding the GFC to 6.6% in 2009. Thus, the QMC model seems to be accurately measuring changes in credit risk at the different quantiles. In contrast, a study by Allen et al. (2011a) shows that credit ratings as used by the Basel Accord standardised capital adequacy model showed negligible changes in credit risk over the GFC, which left banks with inadequate capital to deal with their problem loans. Banks and regulators could therefore do well to use a model such as our QMC model to measure extreme risk rather than relying on static ratings models which do not ratchet up or down in changing economic circumstances.

Figure 1: Financial Industry Betas



β represents the portfolio risk (DD for all industries combined) of each quantile as compared to our portfolio benchmark. The benchmark is the 50% quantile for the total ten year period from 2000 – 2009. So for example, the DD risk at the 95% quantile of our GFC period is 5.44 times that of the 50% quantile for the entire period. The pre-GFC period is the seven years from 2000-2006 and the GFC period is the three years from 2007-2009.

Our second research question is whether relative industry risk changes between the pre-GFC and GFC periods at the extreme quantile. From Table 2 we note several changes. The worst deterioration was experienced by Energy, Utilities, Consumer Discretionary and Financials. The strongest improvement was shown by Telecommunication, IT and Consumer Staples. These results intuitively make sense. Telecommunications and IT experienced extreme volatility in the early pre-GFC period with the bursting of the dotcom bubble. Discretionary products are in less demand in a crisis than staples. The volatility experienced by financial markets and energy prices is well known. We used a Spearman rank correlation coefficient to test for ranking association, and found no significant association between pre-GFC and GFC 95% rankings. This means that extreme risk changes between industries in different economic circumstances; and that those industries that were riskiest pre-GFC are not the same industries that were riskiest during the GFC.

As a robustness check, we apply an alternative approach to measure extreme credit risk of our European Industries. Allen, Powell and Singh (2011) used an historical CVaR

Allen, Boffey & Powell

(Conditional Value at Risk) approach to measure distance to default among industries, which they called CDD. There are two key differences between that approach and the QMC approach used in this study. Firstly, the CDD approach uses actual historical returns to measure asset value fluctuations, whereas QMC uses Monte Carlo simulation. Secondly CDD is based on a simple average of the 5% worst asset returns, whereas we use quantile regression as explained in Section 4. Our QMC approach is a new, more sophisticated approach, but nonetheless historical VaR and CVaR are well known concepts, and thus CVaR model and its CDD credit equivalent provide a good basis for comparison of results. Given the differences in the modelling techniques, exact values are not directly comparable, so we compare industry rankings in Table 2. Whilst the rankings are not exactly the same, due to the modelling differences, we undertook a Spearman Rank correlation test for association in rankings and found the rankings of the two models to be significantly similar at the 99% level of confidence.

Table 2: Relative industry rankings.

	pre-GFC Values		pre-GFC Ranking			GFC Values		GFC Ranking		
	95% Monte Carlo Q	95% CDD	95% Monte Carlo Q	95% CDD	Dsiff. In Rank ²	95% Monte Carlo Q	95% CDD	95% Monte Carlo Q	95% CDD	Dsiff. In Rank ²
Cons. Disc.	1.42	1.31	6	6	0	0.87	0.97	9	8	1
Cons. Stap.	1.43	1.41	5	4	1	1.24	1.34	2	2	0
Energy	1.70	1.47	2	2	0	0.95	1.07	8	5	9
Financials	1.41	1.36	7	5	4	0.81	0.69	10	10	0
Health Care	1.45	1.28	4	7	9	1.20	1.31	3	3	0
Industrials	1.60	1.46	3	3	0	1.08	1.17	4	4	0
IT	1.17	1.07	9	9	0	0.99	1.01	6	7	1
Materials	1.27	1.07	8	8	0	0.98	0.93	7	9	4
Telecomm.	1.15	1.03	10	10	0	1.69	1.80	1	1	0
Utilities	1.95	1.73	1	1	0	1.02	1.07	5	6	1
Total	1.45	1.32			14	1.08	1.14			16
			<i>n</i>		10			<i>n</i>		10
			<i>r</i>		0.915			<i>r</i>		0.903
			<i>t</i>		6.421			<i>t</i>		5.946
			<i>critical value 95%</i>		2.306			<i>critical value 95%</i>		2.306
			<i>critical value 99%</i>		3.355			<i>critical value 99%</i>		3.355
			<i>significance</i>		**			<i>significance</i>		**

The table compares our QMC model (as discussed in Section 4) at the 95% quantile with the CDD model (which calculates DD per equation 1 using average of the worst 5% of asset returns rather than the standard deviation). A ranking of 1 represents the lowest risk and 10 the highest risk. A Spearman rank correlation coefficient (based on differences in ranks²) is used to test for association in ranks between the two models. 95% significance is denoted by * and 99% significance by **.

5. Conclusion

Applying a combination of Monte Carlo Simulation and Quantile Regression to the Merton structural credit model, the study shows significant DD differences between quantiles for all industries studied. Relative industry risk is also shown to change significantly during the circumstances of the GFC period (as described at the start of Section 2) as compared to the pre-GFC period. This has important implications for regulators and lenders. Traditional DD measures or measures based on credit ratings (such as the Basel accord standardised capital adequacy policies) which do not automatically ratchet up or down with changing economic or industry circumstances may not identify extreme risk, and capital and provisioning policies and decisions based on these measures may leave banks short during a downturn. Incorporation of the QMC model into credit risk measurements by lenders and capital adequacy policies by regulators could assist in identifying which industries are most affected during downturn times and may therefore require additional provisions or capital buffers. This study is limited to an analysis of changes in relative industry risk as measured by DD and CDD over the GFC period as compared to pre-GFC. There is potential scope for further research into the economic impacts of the GFC on credit risk through regressing probabilities of default on different macroeconomic aspects of the European economy. The QMC model is also limited to industry risk in European market in this study and there is potential for applying the model to other markets such as the US or Asia, where it has never been used to measure relative industry risk. There is also scope for applying the model to the analysis of individual countries within Europe.

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Allen, Boffey & Powell

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